Centrality Crowding in Asset Prices: A Social-Network Theory of Fragile Price Discovery

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Abstract

This paper develops a concise social-network theory of price discovery. Investors receive noisy private signals about a risky payoff, but form beliefs partly through a social feed that amplifies high-centrality agents. Even when all messages are truthful, the feed can reduce market efficiency because it crowds the price-relevant statistic toward a small set of central signals. In a closed-form equilibrium, the pricing error is governed by a network Herfindahl index. Greater centrality amplification increases price noise, volatility, and subsequent return reversals. With heterogeneous signal precision, centrality-based attention is efficient only when social centrality is aligned with informational accuracy. The model therefore rationalizes why highly connected trading communities may transmit information quickly while also making prices fragile.

Keywords: Social networks; asset pricing; price discovery; centrality; investor attention; market efficiency

JEL classification: D83; D85; G12; G14; G41

1. Introduction

Financial markets increasingly learn through social structures. Retail investors follow influencers; professional investors exchange views through education, employment, board, and geographic networks; and digital platforms rank financial content according to engagement. Survey and empirical evidence has long shown that investor communication and social interaction shape trading decisions (Shiller and Pound, 1989; Hong, Kubik, and Stein, 2004, 2005; Ivkovic and Weisbenner, 2007). More recent work on social finance and news diffusion shows that socially connected investors respond more quickly and strongly to public information (Kuchler and Stroebel, 2021; Hirshleifer, Peng, and Wang, 2025). The standard intuition is that a denser network should accelerate information diffusion. This paper shows a different possibility: a social network can make prices less informative even when every transmitted signal is truthful.

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The mechanism is centrality crowding. If investors give disproportionate attention to high-centrality agents, the market price stops aggregating a diversified set of signals and instead overuses the same central signals. This creates a social redundancy problem. The price reacts strongly to the voices that are easiest to hear, but it need not become more accurate.

The paper builds on four established literatures. First, the pricing environment follows the noisy rational-expectations tradition in which prices aggregate dispersed private information only imperfectly (Grossman, 1976; Grossman and Stiglitz, 1980; Hellwig, 1980; Diamond and Verrecchia, 1981). Second, the mechanism is related to, but distinct from, herding and cascade models: fragility does not arise because agents infer from others' actions and ignore their own signals, but because the same central signal is repeatedly overweighted (Banerjee, 1992; Bikhchandani, Hirshleifer, and Welch, 1992; Hirshleifer and Teoh, 2003). Third, the model extends network-based theories of beliefs, communication, and asset prices by isolating a closed-form centrality-concentration channel (DeMarzo, Vayanos, and Zwiebel, 2003; Ozsoylev and Walden, 2011; Han and Yang, 2013). Fourth, because the distortion operates through scarce investor attention and platform visibility, it connects to limited-attention and media-based price pressure in financial markets (Peng and Xiong, 2006; Tetlock, 2007; Barber and Odean, 2008; Da, Engelberg, and Gao, 2011).

Relative to existing network-finance models, the contribution is deliberately narrow and testable. The paper shows that truthful communication can generate overreaction when social attention is more concentrated than informational precision. This separates the benefit of fast diffusion from the cost of redundant diffusion, and it yields a simple sufficient statistic, the Herfindahl index of centrality-weighted attention, that links network architecture to pricing-error variance, volatility, and return reversals.

2. Networked Attention and Trading

There are $n \geq 2$ risk-averse investors and one risky asset. The asset payoff is

$$\theta \sim N\left(0, \sigma_\theta^2\right).$$

where θ denotes the terminal fundamental value. The prior mean is normalized to zero for tractability. σ_θ^2 is the *ex ante* variance of the fundamental; a larger value means that the asset is intrinsically harder to value before investors observe signals.

Investor i observes a private signal[1]

$$x_i = \theta + \varepsilon_i, \qquad \varepsilon_i \sim N(0, \sigma^2),$$

where x_i is investor i's private signal about θ. ε_i is the idiosyncratic signal error. σ^2 is the common noise variance in the baseline model; lower sigma squared means more precise

[1] Subscripts i and j index investors. Unless otherwise stated, summations run over the n investors in the market. The notation separates fundamental uncertainty, signal noise, social-network centrality, investor attention, and equilibrium pricing weights.

private information. Independence rules out mechanical correlation across signal errors, so any redundancy in prices comes from social attention rather than correlated mistakes.

Let the social network be summarized by a positive centrality vector $c = (c_1, \dots, c_n)$, where $c_i > 0$ and $\sum_i c_i = 1$. For example, c may be eigenvector centrality from a directed communication matrix. A platform or social feed transforms centrality into attention weights

$$r_i(\phi) = \frac{c_i^{\phi}}{\sum_{j=1}^{n} c_j^{\phi}}, \qquad \phi \geq 0.$$

where c_i is investor i's normalized network centrality, with all c_i positive and summing to one. $r_i(\phi)$is the share of market attention allocated to investor i after the feed amplifies centrality. ϕ is the centrality-amplification parameter: ϕ =0 gives equal attention, while larger ϕ shifts attention toward nodes with higher c_i. The denominator ensures that attention weights sum to one.

The parameter ϕ measures centrality amplification. If $\phi = 0$, attention is neutral and $r_i = 1/n$. If ϕ rises, attention shifts toward more central investors.

Investor i forms a reduced-form belief

$$b_i = (1-\rho)x_i + \rho \sum_{j=1}^{n} r_j(\phi) x_j, \qquad \rho \in [0,1],$$

where b_i is investor i's perceived payoff signal used for trading. ρ is the social-attention intensity. When ρ =0, the investor relies only on her own signal x_i. When ρ =1, the investor relies entirely on the centrality-weighted social statistic. The term $\sum_{j=1}^{n} r_j(\phi) x_j$ is the social feed: it averages all truthful signals but gives greater weight to more visible investors.

Investors have CARA preferences and perceive residual payoff variance σ_u^2. Their demand is

$$d_i = \frac{b_i - p}{\gamma \sigma_u^2},$$

where d_i is investor i's demand for the risky asset. p is the equilibrium price. γ is absolute risk aversion under CARA preferences; higher γ makes investors less willing to hold risky positions. σ_u^2 is the residual payoff variance perceived by investors, so $\gamma\sigma_u^2$ is the effective risk penalty in demand.

Proposition 1. Linear equilibrium. The unique competitive equilibrium price is

$$p = \sum_{i=1}^{n} a_i(\rho, \phi) x_i - \lambda Q, \qquad \lambda = \frac{\gamma \sigma_u^2}{n},$$

p is the market-clearing price. $a_i(\rho, \phi)$ is the equilibrium price weight placed on investor i's private signal. λ is the supply-risk-premium coefficient. Q is the exogenous net supply of the risky asset. The risk-premium term λQ affects the level of the price but not the informational content of the price.

where

$$a_i(\rho, \phi) = \frac{1-\rho}{n} + \rho r_i(\phi), \qquad \sum_{i=1}^{n} a_i = 1.$$

The component $\frac{1-\rho}{n}$ captures diversified reliance on all investors' own signals, while $\rho r_i(\phi)$ captures the centrality-weighted social component. The restriction $\sum_{i=1}^{n} a_i = 1$ ensures that the risk-adjusted price is an unbiased weighted average of signals before signal errors are realized.

The relevant pricing error, after removing the risk premium, is

$$\tilde{p} - \theta = \sum_{i=1}^{n} a_i\,(\rho, \phi)\varepsilon_i, \qquad \tilde{p} \equiv p + \lambda Q.$$

$\tilde{p}$ is the risk-premium-adjusted price. The pricing error tilde $\tilde{p} - \theta$ is the difference between the informational component of price and the true fundamental. Because $\sum_{i=1}^{n} a_i = 1$, the fundamental cancels out and only weighted signal errors remain.

Hence

$$M(\rho, \phi) \equiv E[(\tilde{p} - \theta)^2] = \sigma^2 H(\rho, \phi),$$

$M(\rho, \phi)$ is mean squared pricing error, the paper's main measure of price-discovery failure. σ^2 is the variance of each signal error in the equal-precision benchmark. $H(\rho, \phi)$ is the Herfindahl concentration of equilibrium price weights; it converts attention concentration into pricing-error variance.

where

$$H(\rho, \phi) = \sum_{i=1}^{n} a_i\,(\rho, \phi)^2 = \frac{1}{n} + \rho^2 \left[R(\phi) - \frac{1}{n}\right], \qquad R(\phi) = \sum_{i=1}^{n} r_i\,(\phi)^2.$$

$H(\rho, \phi)$ is low when price weights are diversified and high when price weights are concentrated. $R(\phi)$ is the Herfindahl concentration of the social attention vector r($r(\phi)$). The benchmark *1/n* is the lowest possible concentration, obtained under equal weights. The term ρ^2 shows that social concentration matters more when investors place more weight on social information.

Thus, price discovery is governed by a single object: the Herfindahl concentration of price weights.

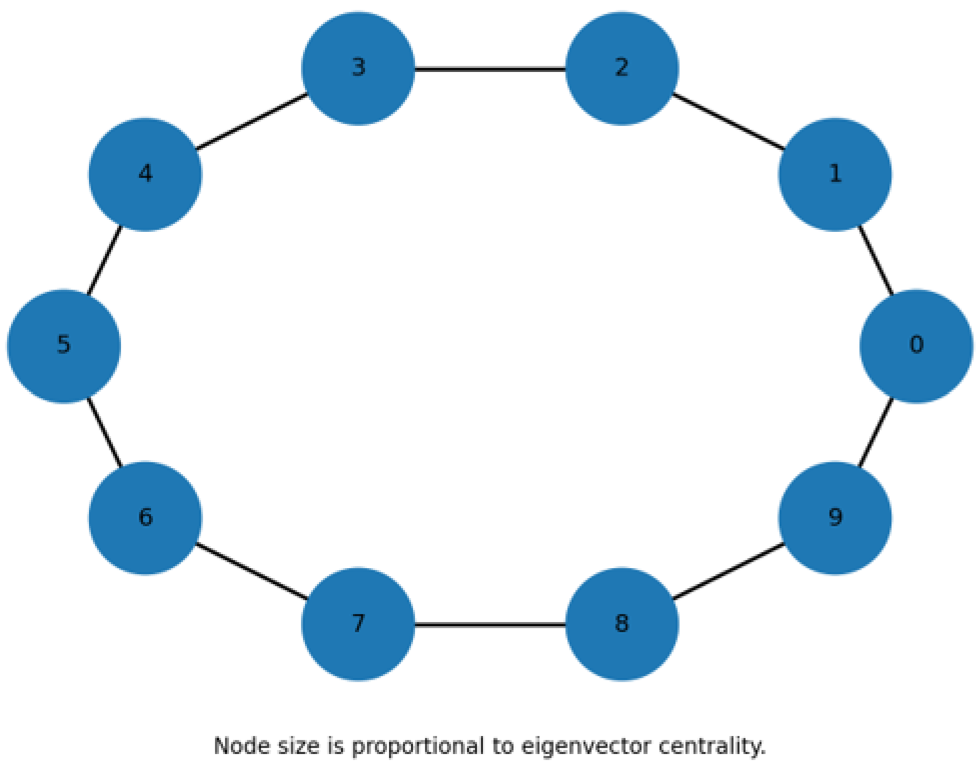
Proof. See Appendix.

3. Network Illustrations

To visualize the mechanism, Figures 1 to 3 plot stylized communication networks. In each figure, node size is proportional to eigenvector centrality, so larger nodes attract disproportionately more social attention when the amplification parameter alpha is positive. The figures help clarify that the model is driven by concentration in the social-weight vector rather than by communication per se.

In Figure 1, in the ring network, agents are locally connected and the centrality vector is nearly uniform. As a result, centrality-based attention remains close to equal weighting, the Herfindahl concentration term stays low, and the market price aggregates a diversified set of signals. This is the benchmark case in which communication does not materially damage price discovery, because no single voice becomes socially dominant.

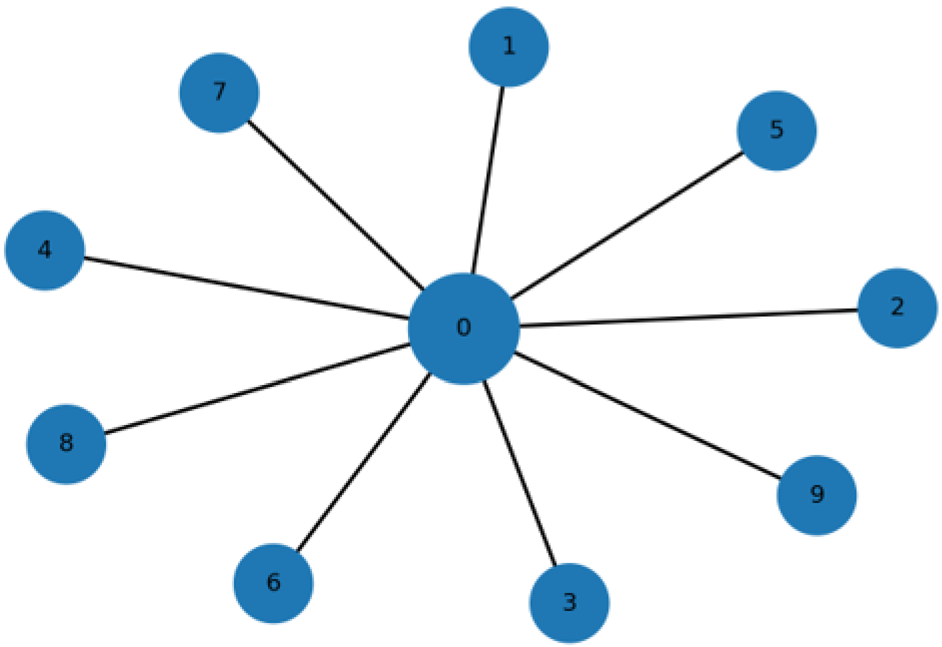
Figure 1: Decentralized ring network



In Figure 2, in the hub-and-spoke network, one investor occupies a dominant social position. When social feeds overweight centrality, attention collapses toward the hub, so many investors repeatedly rely on the same signal. The network therefore generates a high centrality Herfindahl index, exactly the object that increases pricing-error variance in Proposition 2. The figure illustrates the main fragility mechanism: truthful communication can still produce excess volatility and stronger subsequent reversals when the market repeatedly hears the same central voice.

Panel (a) of Figure 3 shows the inefficient case in which the hub is highly central but relatively imprecise, while some peripheral investors possess better signals. In this case, centrality amplification diverts attention toward a noisy source and creates the centrality wedge described in Proposition 4. Panel (b) shows the opposite case: the socially central investor is also the most precise. Then concentration is less harmful and may even approximate the efficient precision-weighted benchmark. The comparison makes clear that the welfare effect of financial social networks depends not simply on connectedness, but on whether visibility is aligned with informational accuracy.

Figure 2: Hub-and-spoke network



The hub has much higher centrality than peripheral nodes.

4. Centrality Crowding

Proposition 2. Centrality amplification reduces price discovery. If the centrality vector is not uniform and $\rho > 0$, then

$$\frac{\partial M(\rho, \phi)}{\partial \phi} > 0.$$

Equivalently, greater amplification of high-centrality agents increases pricing error variance and lowers price informativeness.

The proposition is deliberately stark. Since signals are truthful, the inefficiency does not come from fake news. It comes from repeated use of the same central information. The market aggregates many beliefs, but the beliefs themselves contain a common centrality-weighted component. As ϕ increases, this common component becomes less diversified.

The model also generates a return-predictability implication. Since

$$\tilde{p} = \theta + \eta, \qquad \eta \sim N(0, \sigma^2 H),$$

where η denotes aggregate pricing noise generated by weighted signal errors. Conditional on the fundamental, η is normally distributed with variance $\sigma^2 H$. Thus, H scales how much private-signal noise survives aggregation into the market price.

the conditional expectation of the fundamental is

$$E[\theta \mid \tilde{p}] = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma^2 H(\rho, \phi)} \tilde{p}.$$

This is the Gaussian projection of θ on the risk-adjusted price $\tilde{p}$. The fraction is the signal-extraction coefficient. A larger σ_θ^2 makes price movements more likely to reflect fundamentals; a larger $\sigma^2 H$ makes price movements more likely to reflect noise.

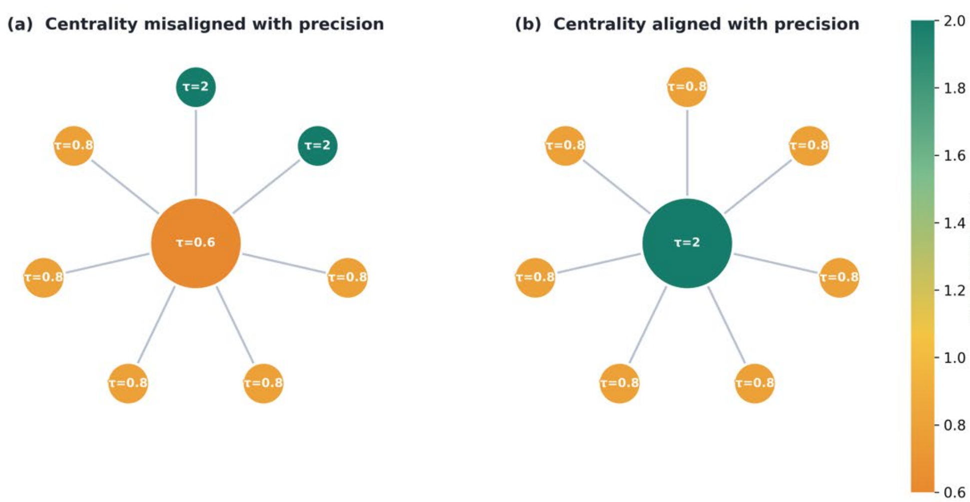
Therefore, the expected correction after an unusually high price is

$$E[\theta - \tilde{p} \mid \tilde{p}] = -\frac{\sigma^2 H(\rho, \phi)}{\sigma_\theta^2 + \sigma^2 H(\rho, \phi)} \tilde{p}.$$

The expression is the expected future correction from the risk-adjusted price back toward the fundamental. For a positive price realization, the correction is negative. The magnitude rises with $\sigma^2 H$, so more concentrated attention predicts stronger return reversals.

Proof. See Appendix.

Figure 3: Centrality-precision alignment



Iode size = eigenvector centrality; colour = signal precision. When the central hub is imprecise (a), amplification creates a costly attention wedge; when it is precise (b), concentration is benign.

Corollary 1. Reversal prediction. For positive price shocks, the expected reversal is larger when centrality amplification is stronger. Formally, for any $\tilde{p} > 0$,

$$|E[\theta - \tilde{p} \mid \tilde{p}]|$$

is increasing in ϕ whenever centrality is heterogeneous and $\rho > 0$. The absolute-value expression measures the expected size of the reversal following a positive risk-adjusted price shock. It is increasing in ϕ because higher ϕ increases H and therefore increases the noise share of the observed price.

This gives the model a direct empirical interpretation: assets whose investor bases are exposed to highly centrality-concentrated communication should display stronger event-day volatility and stronger subsequent reversal, unless central agents are also more precise.

Proof. See Appendix.

5. Social Attention and Network Diversification

The effect of social attention is also transparent. Differentiating the Herfindahl index with respect to ρ gives

$$\frac{\partial H(\rho, \phi)}{\partial \rho} = 2\rho \left[R(\phi) - \frac{1}{n} \right].$$

Proposition 3. Social attention is harmful only when it is concentrated. If attention is neutral, $r_i = 1/n$, then changing ρ has no effect on price discovery. If attention is centrality-concentrated, $R(\phi) > 1/n$, then greater social attention increases price noise.

This result distinguishes communication from concentration. The problem is not that investors talk to each other. The problem is that communication makes many investors rely on the same central voice. A decentralized discussion can preserve diversity, while an engagement-based feed can destroy it.

Proof. See Appendix.

6. Precision, Optimal Attention, and the Centrality Wedge

The baseline assumes equal signal precision. Suppose instead that

$$x_i = \theta + \varepsilon_i, \qquad \mathrm{Var}(\varepsilon_i) = \tau_i^{-1},$$

where $\tau_i > 0$ is investor i's signal precision, a larger τ_i means a more accurate signal. This extension allows informational quality to differ across investors, which is essential for distinguishing useful centrality from purely social visibility.

Consider the pure social-statistic case $\rho = 1$. Then the pricing error is

$$M(r) = \sum_{i=1}^{n} \frac{r_i^2}{\tau_i}.$$

$M(r)$ is the mean squared pricing error when the social statistic receives attention weights r_i. Each term $\frac{r_i^2}{\tau_i}$ shows the cost of putting weight on investor *i*: high weight raises concentration cost, while high precision lowers the cost.

Proposition 4. Precision-weighted attention is efficient. The attention vector that minimizes $M(r)$ subject to $r_i \geq 0$ and $\sum_i r_i = 1$ is

$$r_i^e = \frac{\tau_i}{\sum_{j=1}^{n} \tau_j}.$$

r_i^e is the efficient attention weight assigned to investor *i*. Efficiency requires assigning more attention to more precise investors. The denominator $\sum_{j=1}^{n} \tau_j$ normalizes efficient weights to add up to one.

The efficiency loss of centrality-based attention is

$$M\big(r(\phi)\big) - M(r^e) = \sum_{i=1}^{n} \frac{(r_i(\phi) - r_i^e)^2}{\tau_i} \geq 0.$$

The left side is the extra pricing-error variance caused by using centrality-based attention instead of precision-weighted attention. The right side is a weighted squared distance between social attention $r_i(\phi)$ and efficient attention r_i^e. The nonnegative value formally measures the welfare-relevant centrality-precision misalignment.

Proof. See Appendix.

This proposition clarifies when social networks help. Centrality is not inherently bad. If high-centrality investors are also high-precision investors, centrality-based attention can approximate the efficient allocation of attention. If centrality reflects charisma, engagement, or visibility rather than accuracy, the market inherits an attention wedge:

$$\omega_i(\phi) = r_i(\phi) - r_i^e.$$

$\omega_i(\phi)$ is the centrality wedge for investor *i*. A positive wedge means the investor receives more attention than her signal precision justifies; a negative wedge means the investor is underweighted relative to informational efficiency.

7. Conclusion

This paper develops a compact social-network theory of fragile price discovery. When social attention is tilted toward high-centrality investors, truthful communication can make prices less informative because the market repeatedly reuses a small set of signals rather than aggregating independent information. In equilibrium, the relevant sufficient statistic is the Herfindahl index of centrality-weighted attention, which links centrality amplification to pricing-error variance, volatility, and return reversals. The welfare effect of social networks therefore depends not on connectedness alone, but on whether social visibility is aligned with signal precision.

These results suggest a direct agenda for future empirical work. Centrality concentration in investor-facing networks should predict abnormal event returns, turnover, realized volatility, and subsequent reversals, especially when investors rely heavily on social channels; the effects should be weaker when central agents are also more precise. The policy implication is that platform and market design should preserve informational diversity rather than simply accelerate diffusion. Accuracy-weighted ranking, source diversification, and monitoring of attention concentration can reduce fragile price discovery, while regulators should evaluate the misalignment between social visibility and informational precision as a distinct source of market fragility.

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Appendix. Proofs

Proof of Proposition 1. Market clearing requires that the sum of individual demands equals the aggregate supply, $\sum_{i=1}^{n} d_i = Q$. Substituting the demand function $d_i = \frac{b_i - p}{\gamma \sigma_u^2}$ yields:

$$\sum_{i=1}^{n} \frac{b_i - p}{\gamma \sigma_u^2} = Q$$

Multiplying both sides by $\frac{\gamma \sigma_u^2}{n}$ and rearranging for p provides the equilibrium price:

$$p = \frac{1}{n} \sum_{i=1}^{n} b_i - \frac{\gamma \sigma_u^2}{n} Q$$

Substitute the belief formation equation, $b_i = (1-\rho)x_i + \rho \sum_{j=1}^{n} r_j\,(\phi)x_j$, into the average belief:

$$\frac{1}{n} \sum_{i=1}^{n} b_i = \frac{1}{n} \sum_{i=1}^{n} \left((1-\rho)x_i + \rho \sum_{j=1}^{n} r_j\,(\phi)x_j \right)$$

$$\frac{1}{n} \sum_{i=1}^{n} b_i = \frac{1-\rho}{n} \sum_{i=1}^{n} x_i + \rho \sum_{j=1}^{n} r_j\,(\phi)x_j$$

By factoring out x_i, we can express this as a weighted sum of individual signals:

$$\frac{1}{n} \sum_{i=1}^{n} b_i = \sum_{i=1}^{n} \left(\frac{1-\rho}{n} + \rho r_i(\phi) \right) x_i = \sum_{i=1}^{n} a_i\,(\rho, \phi)x_i$$

This establishes the price formula $p = \sum_{i=1}^{n} a_i\,(\rho, \phi)x_i - \lambda Q$, where $\lambda = \frac{\gamma \sigma_u^2}{n}$. Because the demand is linear and strictly decreasing in \$p\$, this equilibrium is unique.

To find the variance of the pricing error, note that $\sum_{i=1}^{n} a_i = (1-\rho) + \rho \sum_{i=1}^{n} r_i = 1$. The risk-premium-adjusted price simplifies to $\tilde{p} - \theta = \sum_{i=1}^{n} a_i\,\varepsilon_i$. Because the signal errors ε_i are independent, the pricing error variance is $M(\rho, \phi) = \sigma^2 \sum_{i=1}^{n} a_i^2 = \sigma^2 H(\rho, \phi)$. Expanding the Herfindahl index $H(\rho, \phi)$:

$$H(\rho, \phi) = \sum_{i=1}^{n} \left(\frac{1-\rho}{n} + \rho r_i(\phi) \right)^2$$

$$H(\rho, \phi) = \sum_{i=1}^{n} \left(\frac{(1-\rho)^2}{n^2} + \frac{2(1-\rho)\rho}{n} r_i(\phi) + \rho^2 r_i(\phi)^2 \right)$$

Using $\sum r_i(\phi) = 1$ and defining $R(\phi) = \sum r_i(\phi)^2$:

$$H(\rho, \phi) = \frac{(1-\rho)^2}{n} + \frac{2(1-\rho)\rho}{n} + \rho^2 R(\phi)$$

$$H(\rho, \phi) = \frac{1 - 2\rho + \rho^2 + 2\rho - 2\rho^2}{n} + \rho^2 R(\phi) = \frac{1-\rho^2}{n} + \rho^2 R(\phi) = \frac{1}{n} + \rho^2 \left(R(\phi) - \frac{1}{n}\right)$$

Proof of Proposition 2. Let $L_i = \log c_i$, such that $c_i^\phi = \exp(\phi L_i)$. The attention weight is $r_i(\phi) = \frac{\exp(\phi L_i)}{\sum_{j=1}^n \exp(\phi L_j)}$. Differentiating r_i with respect to ϕ using the quotient rule yields:

$$\frac{\partial r_i}{\partial \phi} = \frac{L_i \exp(\phi L_i) \sum_j \exp(\phi L_j) - \exp(\phi L_i) \sum_j L_j \exp(\phi L_j)}{\left(\sum_j \exp(\phi L_j)\right)^2}$$

$$\frac{\partial r_i}{\partial \phi} = r_i L_i - r_i \sum_{j=1}^n r_j L_j = r_i (L_i - E_r[L])$$

Now, differentiate the concentration index $R(\phi) = \sum_{i=1}^n r_i(\phi)^2$ with respect to ϕ:

$$R'(\phi) = \sum_{i=1}^n 2 r_i \frac{\partial r_i}{\partial \phi} = 2 \sum_{i=1}^n r_i^2 (L_i - E_r[L])$$

$$R'(\phi) = 2\left(\sum_{i=1}^n r_i^2 L_i - E_r[L] \sum_{i=1}^n r_i^2\right) = 2R(\phi)\left(\sum_{i=1}^n \frac{r_i^2}{R(\phi)} L_i - E_r[L]\right)$$

Define a new probability measure $s_i = \frac{r_i^2}{R(\phi)}$. Because r_i is strictly increasing in L_i, the distribution s places strictly more relative weight on higher values of L_i compared to the distribution r. Thus, s strictly first-order stochastically dominates r whenever c (and thus L) is non-uniform, ensuring that $E_s[L] > E_r[L]$. Therefore:

$$R'(\phi) = 2R(\phi)\{E_s[L] - E_r[L]\} > 0$$

Because $H(\rho, \phi) = \frac{1}{n} + \rho^2 \left(R(\phi) - \frac{1}{n}\right)$, its derivative is $H'(\phi) = \rho^2 R'(\phi)$, which is strictly positive for $\rho > 0$. Consequently, $M(\rho, \phi) = \sigma^2 H(\rho, \phi)$ is strictly increasing in ϕ.

Proof of Corollary 1. Assume without loss of generality that Q is observed or the price is risk-premium adjusted, so we project θ onto $\tilde{p} = \theta + \eta$, where $\eta = \sum_{i=1}^n a_i \varepsilon_i \sim N(0, \sigma^2 H)$. By the standard projection theorem for joint normal variables:

$$E[\theta \mid \tilde{p}] = \frac{Cov(\theta, \tilde{p})}{Var(\tilde{p})}\tilde{p}$$

Since $Cov(\theta, \theta + \eta) = \sigma_\theta^2$ and $Var(\theta + \eta) = \sigma_\theta^2 + \sigma^2 H$, we have:

$$E[\theta \mid \tilde{p}] = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma^2 H}\tilde{p}$$

The expected reversal (correction) is:

$$E[\theta - \tilde{p} \mid \tilde{p}] = \left(\frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma^2 H} - 1\right)\tilde{p} = \frac{-\sigma^2 H}{\sigma_\theta^2 + \sigma^2 H}\tilde{p}$$

For any positive price shock ($\tilde{p} > 0$), the absolute magnitude of this expected reversal is $\left(\frac{\sigma^2 H}{\sigma_\theta^2 + \sigma^2 H}\right)\tilde{p}$. Taking the derivative of the fractional term with respect to H yields a strictly positive result. From Proposition 2, H is strictly increasing in ϕ, meaning the expected reversal magnitude strictly increases with centrality amplification ϕ.

Proof of Proposition 3. Differentiating the Herfindahl index $H(\rho, \phi)$ with respect to social attention intensity ρ yields:

$$\frac{\partial H}{\partial \rho} = 2\rho\left(R(\phi) - \frac{1}{n}\right)$$

By the Cauchy-Schwarz inequality, $R(\phi) = \sum_{i=1}^{n} r_i^2 \geq \frac{(\sum r_i)^2}{n} = \frac{1}{n}$. The derivative $\frac{\partial H}{\partial \rho}$ is therefore strictly positive whenever attention is not uniform ($R(\phi) > 1/n$) and $\rho > 0$. If attention is perfectly uniform ($r_i = 1/n$), $R(\phi) = 1/n$, and the derivative is zero, meaning ρ does not impact pricing error.

Proof of Proposition 4. To find the efficient attention allocation, we minimize the pricing error variance $M_r = \sum_{i=1}^{n} \frac{r_i^2}{\tau_i}$ subject to $\sum_{i=1}^{n} r_i = 1$. The Lagrangian is:

$$\mathcal{L} = \sum_{i=1}^{n} \frac{r_i^2}{\tau_i} - \mu\left(\sum_{i=1}^{n} r_i - 1\right)$$

The first-order condition (F.O.C) with respect to r_i is:

$$\frac{2r_i}{\tau_i} = \mu \Longrightarrow r_i = \frac{\mu \tau_i}{2}$$

Substitute this into the constraint $\sum r_i = 1$:

$$\sum_{i=1}^{n} \frac{\mu\tau_i}{2} = 1 \Longrightarrow \frac{\mu}{2}\sum_{j=1}^{n} \tau_j = 1 \Longrightarrow \mu = \frac{2}{\sum_{j=1}^{n} \tau_j}$$

Substituting μ back into the F.O.C provides the efficient weights $r_i^e = \frac{\tau_i}{\sum_{j=1}^{n} \tau_j}$. Because the objective function is strictly convex, this global minimum is unique.

To establish the decomposition of the efficiency loss:

$$M_r - M_r^e = \sum_{i=1}^{n} \frac{r_i^2}{\tau_i} - \sum_{i=1}^{n} \frac{(r_i^e)^2}{\tau_i}$$

Note that $\frac{r_i^e}{\tau_i} = \frac{1}{\sum \tau_j}$ for all i. We can expand the squared wedge term:

$$\sum_{i=1}^{n} \frac{(r_i - r_i^e)^2}{\tau_i} = \sum_{i=1}^{n} \frac{r_i^2}{\tau_i} - 2\sum_{i=1}^{n} \frac{r_i r_i^e}{\tau_i} + \sum_{i=1}^{n} \frac{(r_i^e)^2}{\tau_i}$$

Evaluate the cross-term:

$$-2\sum_{i=1}^{n} r_i \left(\frac{r_i^e}{\tau_i}\right) = -2\left(\frac{1}{\sum \tau_j}\right)\sum_{i=1}^{n} r_i = -\frac{2}{\sum \tau_j}$$

Evaluate the final term:

$$\sum_{i=1}^{n} r_i^e \left(\frac{r_i^e}{\tau_i}\right) = \left(\frac{1}{\sum \tau_j}\right)\sum_{i=1}^{n} r_i^e = \frac{1}{\sum \tau_j}$$

Combining these gives:

$$\sum_{i=1}^{n} \frac{(r_i - r_i^e)^2}{\tau_i} = \sum_{i=1}^{n} \frac{r_i^2}{\tau_i} - \frac{1}{\sum \tau_j} = M_r - M_r^e$$

Since the left side is a sum of squared terms, $M_r(\phi) - M_r^e \geq 0$, confirming the efficiency loss.